


Analysis of datasheets of brushed DC motors

Maxon motors: <https://www.maxongroup.com/en/drives-and-systems/brushed-dc-motors>

Case study: datasheet for Faulhaber 4.2 mNm DC motors


FAULHABER

DC-Micromotors

4,2 mNm

Precious Metal Commutation

For combination with
Gearheads:
15A, 16/7, 16A
Encoders:
IE2-1024, IE2-16

Series 1724 ... SR

	1724 T	003 SR	006 SR	012 SR	018 SR	024 SR				
1 Nominal voltage	U_N	3	6	12	18	24	V			
2 Terminal resistance	R	0,78	3,41	16,2	32,1	54,6	Ω			
3 Output power	$P_2 \text{ max.}$	2,83	2,58	2,17	2,47	2,58	W			
4 Efficiency, max.	$\eta \text{ max.}$	82	81	80	81	81	%			
5 No-load speed	n_0	8 200	8 600	7 900	8 400	8 600	rpm			
6 No-load current (with shaft \varnothing 1,5 mm)	I_0	0,038	0,02	0,009	0,006	0,005	A			
7 Stall torque	M_H	13,2	11,5	10,5	11,2	11,5	mNm			
8 Friction torque	M_R	0,13	0,13	0,13	0,12	0,13	mNm			
9 Speed constant	k_n	2 760	1 450	666	472	362	rpm/V			
10 Back-EMF constant	k_E	0,362	0,69	1,5	2,12	2,76	mV/rpm			
11 Torque constant	k_M	3,46	6,59	14,3	20,2	26,3	mNm/A			
12 Current constant	k_i	0,289	0,152	0,07	0,049	0,038	A/mNm			
13 Slope of n-M curve	$\Delta n / \Delta M$	621	748	752	750	748	rpm/mNm			
14 Rotor inductance	L	21	75	360	710	1 200	μH			
15 Mechanical time constant	τ_m	8	8	8	8	8	ms			
16 Rotor inertia	J	1,2	1	1	1	1	gcm ²			
17 Angular acceleration	$\alpha \text{ max.}$	110	110	100	100	100	$\cdot 10^3 \text{ rad/s}^2$			
18 Thermal resistance	R_{th1} / R_{th2}	4 / 24,5					K/W			
19 Thermal time constant	τ_{w1} / τ_{w2}	2,6 / 270					s			
20 Operating temperature range:		-30 ... +85 (optional version -55 ... +125)					°C			
– motor										
– rotor, max. permissible		+125					°C			
21 Shaft bearings		sintered bearings		ball bearings		ball bearings, preloaded				
22 Shaft load max.:		(standard)		(optional version)		(optional version)				
– with shaft diameter		1,5		1,5		1,5	mm			
– radial at 3 000 rpm (3 mm from bearing)		1,2		5		5	N			
– axial at 3 000 rpm		0,2		0,5		0,5	N			
– axial at standstill		20		10		10	N			
23 Shaft play										
– radial	\leq	0,03		0,015		0,015	mm			
– axial	\leq	0,2		0,2		0	mm			
24 Housing material		steel, black coated								
25 Weight		27					g			
26 Direction of rotation		clockwise, viewed from the front face								
Recommended values - mathematically independent of each other										
27 Speed up to	$n \text{ max.}$	8 000	8 000	8 000	8 000	8 000	rpm			
28 Torque up to	$M \text{ max.}$	4,2	4,2	4,2	4,2	4,2	mNm			

General model for a brushed DC motor with a stator “s” and armature “a” (rotor) coils:

$$\begin{cases} V_a(t) = R_a \times I_a(t) + L_a \times \frac{\partial I_a(t)}{\partial t} + K_b \times I_s(t) \times \frac{\partial \theta(t)}{\partial t} \\ V_s(t) = R_s \times I_s(t) + L_s \times \frac{\partial I_s(t)}{\partial t} \\ J \frac{\partial^2 \theta(t)}{\partial t^2} + B \frac{\partial \theta(t)}{\partial t} + k\theta(t) = T_{mag}(t) + T_{ex}(t) \\ T_{mag}(t) = K_T \times I_s(t) \times I_a(t) \\ K_b = const \\ K_T = const \end{cases} \quad (1)$$

where:

T_{mag} – magnetic torque

V_a – armature voltage

V_s – stator voltage

I_a – armature current

I_s – stator current

R_a – armature resistance

R_s – stator resistance

L_a – armature self-inductance

L_s – stator self-inductance

J – moment of inertia

$\omega = \frac{\partial \theta(t)}{\partial t}$ – angular speed

$\alpha = \frac{\partial^2 \theta(t)}{\partial t^2}$ – angular acceleration

$B \frac{\partial \theta(t)}{\partial t}$ – torque proportional to the angular speed and directed against T_{mag}

$k\theta(t)$ – Hook's torque directed against T_{mag}

For a permanent magnet DC motor, there are no parameters associated with the stator:

$$\begin{cases} V_a(t) = R_a \times I_a(t) + L_a \times \frac{\partial I_a(t)}{\partial t} + K_F \times \frac{\partial \theta(t)}{\partial t} \\ J \frac{\partial^2 \theta(t)}{\partial t^2} + B \frac{\partial \theta(t)}{\partial t} + k\theta(t) = T_{mag}(t) + T_{ex}(t) + T_f \\ T_{mag}(t) = K_M \times I_a(t) \end{cases} \quad (2)$$

where T_f is the friction torque directed against $T_{mag}(t) + T_{ex}(t)$.

For a steady rotation, we have:

$$\omega = const$$

$$\alpha = 0 \quad (3)$$

$$V_a = const$$

$$\frac{\partial I_a(t)}{\partial t} = 0$$

Then:

$$V_a = R_a \times I_a + K_F \times \omega \quad (4)$$

$$T_{mag} = K_M \times I_a \quad (5)$$

Expressing I_a through T_{mag} in (5) and substituting it to (4), we obtain:

$$I_a = \frac{T_{mag}}{K_M} \quad (6)$$

$$T_{mag} = \frac{V_a K_M}{R_a} - \frac{K_F K_M}{R_a} \times \omega \quad (\text{speed-to-torque equation}) \quad (7)$$

$$\omega = \frac{V_a}{K_F} - \frac{R_a}{K_F K_M} \times T_{mag} \quad (\text{torque-to-speed equation}) \quad (8)$$

Both Eqs. (7) and (8) are the straight lines with negative slopes. From Eqs. (7) and (8), we can derive the maximum torque and speed:

$$\max T_{mag} \big|_{\omega=0} = \frac{V_a K_M}{R_a} \quad (9)$$

$$\max \omega \big|_{T_{mag}=T_f} = \frac{V_a}{K_F} - \frac{T_f R_a}{K_F K_M} \quad (10)$$

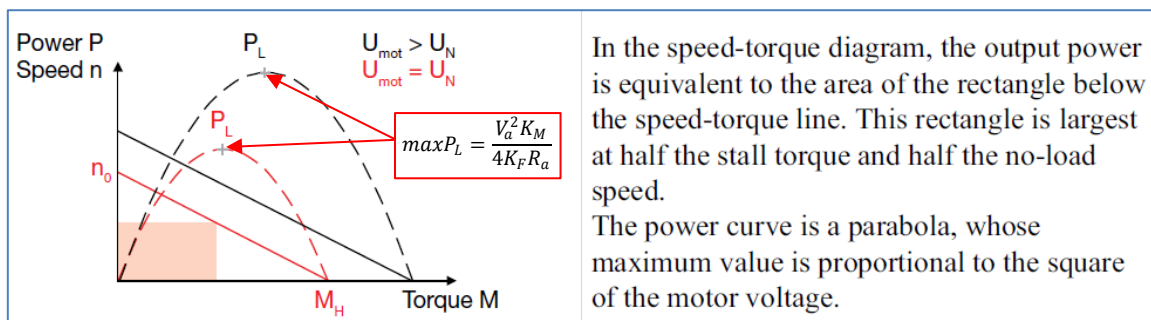
If $B = T_f \equiv 0$ (no frictions) and $T_{ex} = 0$, then $T_{mag} = 0$ and $I_a = 0$. In practice, I_a may be very small for a free rotation, but it is never zero due to a residual friction that always presents in a motor.

The mechanical power generated by the motor to compensate for the load torque is

$P_L = T_{mag} \times \omega$, where $\omega = \frac{V_a}{K_F} - \frac{R_a}{K_F K_M} \times T_{mag}$ in Eq. (8):

$$P_L(T_{mag}) = \left(\frac{V_a}{K_F} - \frac{T_{mag} R_a}{K_F K_M} \right) \times T_{mag} \quad (11)$$

where $T_{mag} \in \left[T_f, \frac{V_a K_M}{R_a} \right]$. This is a quadratic function shown below.



U_N – nominal armature voltage, n – angular speed in [Maxon Formulae Handbook](#).

The maximum mechanical power P_L is found from the condition $\frac{\partial P_L(T_{mag})}{\partial T_{mag}} = 0$:

$$\begin{aligned} \frac{\partial P_L(T_{mag})}{\partial T_{mag}} = 0 &\rightarrow T_{mag} = \frac{V_a K_M}{2R_a} \rightarrow \omega = \frac{V_a}{2K_F} \\ \max P_L &= \frac{V_a^2 K_M}{4K_F R_a} \approx \frac{V_a^2}{4R_a} \end{aligned} \quad (12)$$

The motor efficiency η is defined as the ratio of the mechanical power $(T_{mag} - T_f) \times \omega$ to the electrical power $P_E = V_a \times I_a$ supplied to the motor. Using Eqs. (6) and (8), we obtain:

$$\eta(T_{mag}) = \frac{(T_{mag} - T_f) \times \omega}{V_a \times I_a} = \frac{K_M}{K_F} \left(1 - \frac{R_a}{V_a K_M} T_{mag}\right) \times \left(1 - \frac{T_f}{T_{mag}}\right) \quad (13)$$

The maximum efficiency is found from the condition $\frac{\partial \eta(T_{mag})}{\partial T_{mag}} = 0$:

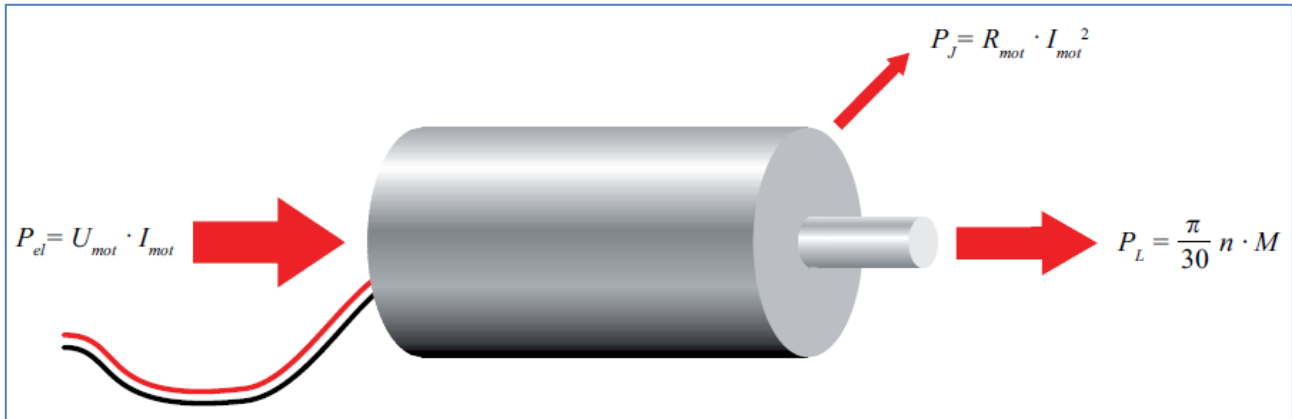
$$\frac{\partial \eta}{\partial T_{mag}} = 0 \rightarrow T_{mag} = \sqrt{\frac{V_a K_M T_f}{R_a}} = \sqrt{\max T_{mag} \times T_f} \quad (14)$$

Substituting this value of T_{mag} to Eq. (13), we obtain the maximum efficiency:

$$\max \eta = \frac{K_M}{K_F} \left(1 - \sqrt{\frac{R_a T_f}{V_a K_M}}\right)^2 = \frac{K_M}{K_F} \left(1 - \sqrt{\frac{I_a^{no-load}}{\max I_a}}\right)^2 \quad (15)$$

where $I_a^{no-load}$ is the no-load armature current and $\max I_a = \frac{V_a}{R_a}$ is the maximum armature current.

Power conservation law for the motor: **Electrical power = Joule Loss + Mechanical Work**



In [Maxon's Formulae Handbook](#): $\frac{\pi}{30} \times n$ is the angular speed n in rpm recalculated into rad/s. $I_{mot} = I_a$, $U_{mot} = V_a$, $R_{mot} = R_a$, and $M = T_{mag}$. **Due to the absence of a magnetic core in the rotor, Maxon motors have a negligible magnetic loss.**

Let us extract the model parameters from the datasheet for Faulhaber 006 SR:

- (2) Terminal (armature) resistance $R_a = 3.41 \, \Omega$
- (14) Rotor's (armature) inductance $L_a = 7.5 \times 10^{-5} \, \text{H}$
- (10) Back-EMF constant $K_F = 0.69 \, \text{mV/rpm}$ or $K_F = 0.69 \times 10^{-3} / (2\pi/60) = 6.589 \times 10^{-3} \, \text{V}\times\text{s/rad}$
- (16) Rotor's moment of inertia $J = 1 \, \text{g}\times\text{cm}^2$ or $J = 10^{-7} \, \text{kg}\times\text{m}^2$
- (11) Torque constant $K_M = 6.59 \times 10^{-3} \, \text{N}\times\text{m/A}$
- (8) Friction torque $T_f = 1.3 \times 10^{-4} \, \text{N}\times\text{m}$

Note that K_F and K_M have the same values but different dimensions.

Checking the validity of the model

1. Calculate the slope $\Delta\omega/\Delta T$ and compare with (13) in the datasheet:

The graph of $\omega(T_{mag})$ in Eq. (8) is a straight line with the slope $\Delta\omega/\Delta T = R_a/(K_F K_M) \approx 78533 \, \text{rad}/(\text{s}\times\text{N}\times\text{m})$ – theoretical value. The experimental slope in (13) is $748 \, \text{rpm}/(\text{mN}\times\text{m}) = 748 \times \pi \times 10^3 / 30 \, \text{rad}/(\text{s}\times\text{N}\times\text{m}) \approx 78330 \, \text{rad}/(\text{s}\times\text{N}\times\text{m})$. Good agreement with (13) in the datasheet.

2. Calculate the no-load speed $\max \omega$ in Eq. (10) and compare it with (5) in the datasheet:

$\max \omega|_{T_{mag}=T_f} = \frac{V_a}{K_F} - \frac{T_f R_a}{K_F K_M} \approx 900.4$, where $V_a = 6 \, \text{V}$. The experimental $\max \omega$ is $8600 \, \text{rpm} = 8600 \times \pi / 30 \, \text{rad/s} \approx 900.6 \, \text{rad/s}$. Good agreement with (5) in the datasheet.

3. Calculate $\max T_{mag}$ in Eq. (9) and compare it with (7) in the datasheet:

$\max T_{mag}|_{\omega=0} = \frac{V_a K_M}{R_a} \approx 1.1595 \times 10^{-2} \, \text{N}\times\text{m}$, where $V_a = 6 \, \text{V}$. The experimental $\max T_{mag}$ (stall torque) is $11.5 \, \text{mN}\times\text{m} = 1.15 \times 10^{-2} \, \text{N}\times\text{m}$. Good agreement with (7) in the datasheet.

4. Calculate the speed constant and compare it with (9) in the datasheet:

Using Eq. (10), $\max \omega \approx \frac{V_a}{K_F}$. So, the speed constant is $\frac{1}{K_F} \approx 151.7681 \, \text{rad}/(\text{V}\times\text{s})$. The experimental value is $1450 \, \text{rpm/V} = 1450 \times \pi / 30 \, \text{rad}/(\text{V}\times\text{s}) \approx 151.8436 \, \text{rad}/(\text{V}\times\text{s})$. Good agreement with (9) in the datasheet.

5. Express the friction torque (8) through other constants in the datasheet:

For the free rotation (no-load) with $T_{mag} \approx |T_f|$. On the other hand, $T_{mag} = K_M \times I_a$. From these two equations, we obtain for the friction torque: $T_f = K_M \times I_a^{no-load}$. Using the no-load current (6) from the datasheet $I_a^{no-load} = 2 \times 10^{-2}$ A and $K_M = 6.59 \times 10^{-3}$ N×m/A, we obtain: $T_f \approx 6.59 \times 10^{-3} \times 2 \times 10^{-2} = 1.318 \times 10^{-4}$ N×m. Good agreement with (8) in the datasheet.

6. Express the current constant (12) through other constants in the datasheet:

Using $I_a = \frac{T_{mag}}{K_M}$ in Eq. (6) and $K_M = 6.59 \times 10^{-3}$ N×m/A from (a), we can calculate the current constant $\frac{1}{K_M} \approx 151.7451$ A/(N×m). Good agreement with (12) in the datasheet.

7. Express the angular acceleration (17) through other constants in the datasheet:

Using the mechanical equation $J \frac{\partial^2 \theta(t)}{\partial t^2} + B \frac{\partial \theta(t)}{\partial t} + k\theta(t) = T_{mag}(t) + T_{ex}(t) + T_f$, for free rotation we obtain: $J\alpha(t) = T_{mag}(t) + T_f$, where $\alpha(t) = \frac{\partial^2 \theta(t)}{\partial t^2}$ is the angular acceleration and T_f is directed against $T_{mag}(t)$. In the beginning of motion, when the magnetic torque takes its maximum value $\frac{V_a K_M}{R_a}$, we obtain: $\alpha = \frac{\max T_{mag} - T_f}{J} = \frac{1.15 \times 10^{-2} - 1.318 \times 10^{-4}}{10^{-7}} = 113682$ rad/s². Here, $V_a = 6$ V. Good agreement with (17) in the datasheet.

8. Calculate the maximum motor efficiency in Eq. (15) and compare it with (4) in the datasheet:

$\max \eta = \frac{K_M}{K_F} \left(1 - \sqrt{\frac{I_a^{no-load}}{\max I_a}} \right)^2 \approx 0.8$ (80%), where $I_a^{no-load} = 0.02$ A is the no-load armature current (see (6) in the datasheet) and $\max I_a = \frac{V_a}{R_a} = \frac{6}{3.41} \approx 1.76$ A. Good agreement with (4) in the datasheet.

9. Calculate the maximum motor power in Eq. (12) and compare it with (3) in the datasheet:

$$\max P_L = \frac{V_a^2 K_M}{4 K_F R_a} \approx 2.64 \text{ W. Good agreement with (3) in the datasheet.}$$

$$\max P_L = 2.64 \text{ W} \rightarrow T_{mag} = \frac{V_a K_M}{2 R_a} \approx 5.8 \text{ mN}\times\text{m} \quad \omega = \frac{V_a}{2 K_F} \approx 455 \text{ rad/s}$$

$$\text{Efficiency } \eta(T_{mag}) \approx 0.5 \text{ (50\%)}$$

Note that the recommended torque (28) from the datasheet is 4.2 mN×m, which is below the torque required for the maximum mechanical power. It is not clear from what criteria this torque value was selected. Maybe 50% efficiency?

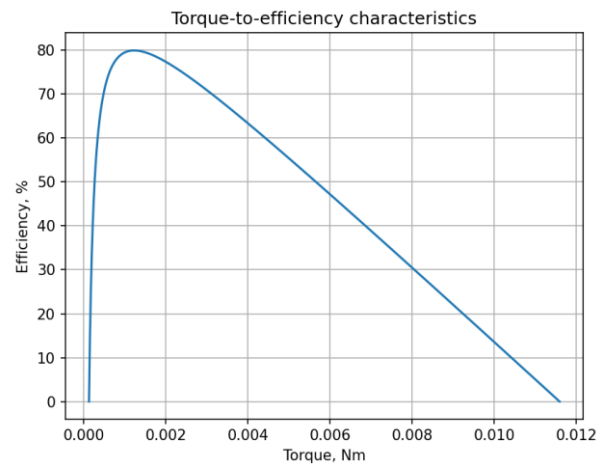
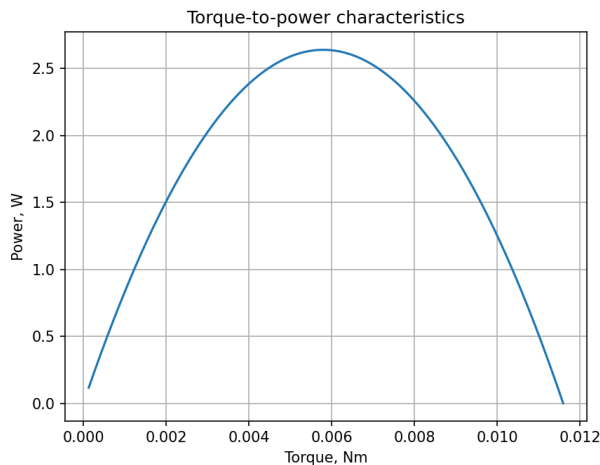
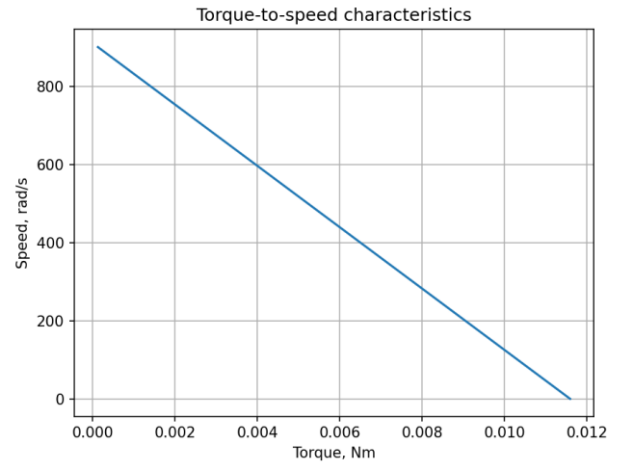
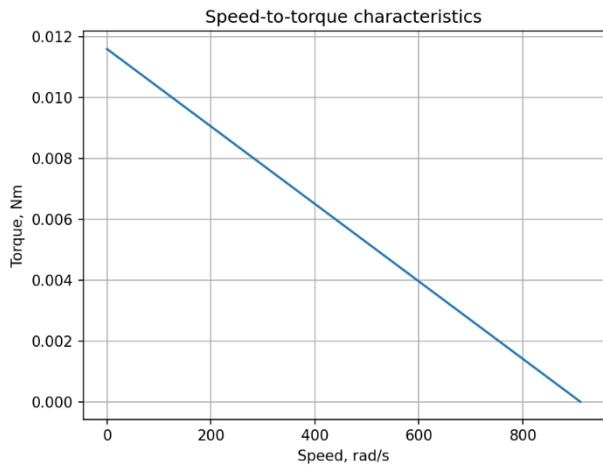
Motor solver in Python: https://github.com/DmitriyMakhnovskiy/Brushed_DC_motor_solver

Output files: Speed-to-torque_characteristics.csv, Torque-to-speed_characteristics.csv, Torque-to-power_characteristics.csv, Torque-to-efficiency_characteristics.csv.

Output parameters (printed to console): Maximum speed, Maximum torque, Maximum efficiency, Maximum mechanical power, No-load armature current, Torque-to-current coefficient, Voltage-to-speed coefficient (no load), dw/dT slope, Maximum angular acceleration (no load).

```
1. # Motor constants used in the model:
2. Va = 6.0 # Armature voltage, V
3. Ra = 3.41 # Armature resistance, Ohms
4. La = 7.5e-5 # Armature inductance, H
5. KF = 6.589e-3 # Back-EMF constant, Vs/rad
6. J = 1.0e-7 # Rotor moment of inertia, kgm^2
7. KM = 6.59e-3 # Torque constant, Nm/A
8. Tf = 1.3e-4 # Friction torque, Nm
```

```
1. Output parameters:
2. Maximum speed = 9.004e+02 rad/s
3. Maximum torque = 1.160e-02 Nm
4. Maximum efficiency = 7.996e+01 %
5. Maximum mechanical power = 2.640e+00 W
6. No-load armature current = 1.973e-02 A
7. Torque-to-current coefficient = 1.517e+02 A/(Nm)
8. Voltage-to-speed coefficient (no load) = 1.518e+02 rad/(Vs)
9. dw/dT slope = 7.853e+04 rad/(sNm)
10. Maximum angular acceleration (no load) = 1.147e+05 rad/s^2
```



main.py:

```

1. #
2. # Solver for brushed DC motors
3. # Further reading: https://support.maxongroup.com/hc/en-us/articles/360001900933-Formulae-Handbook
4. #
5. # Dr. Dmitriy Makhnovskiy, City College Plymouth, England
6. # 30.03.2024
7. #
8.
9. import matplotlib.pyplot as plt
10. import csv
11.
12. # Motor constants used in the model:
13. Va = 6.0 # Armature voltage, V
14. Ra = 3.41 # Armature resistance, Ohms
15. La = 7.5e-5 # Armature inductance, H
16. Kf = 6.589e-3 # Back-EMF constant, Vs/rad
17. J = 1.0e-7 # Rotor moment of inertia, kgm^2
18. Km = 6.59e-3 # Torque constant, Nm/A
19. Tf = 1.3e-4 # Friction torque, Nm
20. N = 1000 # Number of points in the graph
21.
22. # Characteristic parameters:
23. maxw0 = Va / Kf # Maximum angular speed without the friction torque, rad/s
24. maxw = Va / Kf - (Tf * Ra) / (Kf * Km) # Maximum angular speed with the friction torque, rad/s
25. maxTmag = (Va * Km) / Ra # Maximum magnetic torque, Nm
26. maxh = (Km / Kf) * (1.0 - ((Ra * Tf) / (Va * Km))**0.5)**2 # Maximum efficiency
27. maxPL = (Va**2 * Km) / (4.0 * Kf * Ra) # Maximum power, W
28. I0 = Tf / Km # No-load armature current, A

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29. TIa = 1.0 / KM # Torque-to-current coefficient, A/(Nm)
30. Vaw = 1.0 / KF # Voltage-to-speed coefficient (no load), rad/(Vs)
31. dwdT = Ra / (KF * KM) # dw/dT slope, rad/(sNm)
32. alfa = (maxTmag - Tf) / J # Maximum angular acceleration (no load), rad/s^2
33. print('Output parameters:')
34. print('Maximum speed = ', format(maxw, ".3e"), ' rad/s' )
35. print('Maximum torque = ', format(maxTmag, ".3e"), ' Nm')
36. print('Maximum efficiency = ', format(maxh * 100, ".3e"), '%')
37. print('Maximum mechanical power = ', format(maxPL, ".3e"), ' W')
38. print('No-load armature current = ', format(I0, ".3e"), ' A')
39. print('Torque-to-current coefficient = ', format(TIa, ".3e"), ' A/(Nm)')
40. print('Voltage-to-speed coefficient (no load) = ', format(Vaw, ".3e"), ' rad/(Vs)')
41. print('dw/dT slope = ', format(dwdT, ".3e"), ' rad/(sNm)')
42. print('Maximum angular acceleration (no load) = ', format(alfa, ".3e"), ' rad/s^2')
43.
44. # Arrays for graphs
45. wx = [0.0 + maxw0 * i / (N - 1) for i in range(N)] # Angular speed array
46. Tx = [Tf + (maxTmag - Tf) * i / (N - 1) for i in range(N)] # Magnetic torque array
47. T_w = [(Va * KM) / Ra - (KF * KM * w) / Ra for w in wx] # Speed-to-torque characteristics
48. w_T = [Va / KF - (Ra * T) / (KF * KM) for T in Tx] # Torque-to-speed characteristics
49. PL_T = [(Va / KF - (T * Ra) / (KF * KM)) * T for T in Tx] # Torque-to-power characteristics
50. h_T = [(KM / KF) * (1.0 - (Ra * T) / (Va * KM)) * (1.0 - Tf / T) for T in Tx] # Torque-to-efficiency
characteristics
51.
52. # Function to plot graphs and write arrays to files
53. def plot_and_save_data(x, y, x_label, y_label, graph_title, filename):
54.     # Plotting the graph
55.     plt.plot(x, y)
56.     plt.xlabel(x_label)
57.     plt.ylabel(y_label)
58.     plt.title(graph_title)
59.
60.     # Adding detailed grid
61.     plt.grid(True)
62.
63.     # Saving data to CSV file
64.     with open(filename, 'w', newline='') as csvfile:
65.         csv_writer = csv.writer(csvfile)
66.         csv_writer.writerow([x_label, y_label]) # Write header
67.         for i in range(len(x)):
68.             csv_writer.writerow([x[i], y[i]])
69.
70.     # Displaying the plot
71.     plt.show()
72.
73. x_label = 'Speed, rad/s'
74. y_label = 'Torque, Nm'
75. graph_title = 'Speed-to-torque characteristics'
76. filename = 'Speed-to-torque_characteristics.csv'
77. plot_and_save_data(wx, T_w, x_label, y_label, graph_title, filename)
78.
79. x_label = 'Torque, Nm'
80. y_label = 'Speed, rad/s'
81. graph_title = 'Torque-to-speed characteristics'
82. filename = 'Torque-to-speed_characteristics.csv'
83. plot_and_save_data(Tx, w_T, x_label, y_label, graph_title, filename)
84.
85. x_label = 'Torque, Nm'
86. y_label = 'Power, W'
87. graph_title = 'Torque-to-power characteristics'
88. filename = 'Torque-to-power_characteristics.csv'
89. plot_and_save_data(Tx, PL_T, x_label, y_label, graph_title, filename)
90.
91. x_label = 'Torque, Nm'
92. y_label = 'Efficiency, %'
93. graph_title = 'Torque-to-efficiency characteristics'
94. filename = 'Torque-to-efficiency_characteristics.csv'
95. h_T = [h_T * 100.0 for h_T in h_T] # Transferring to %
96. plot_and_save_data(Tx, h_T, x_label, y_label, graph_title, filename)

```